

# HUMIDITY CONVERSION FORMULAS

Calculation formulas for humidity



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## 1. Relative humidity definitions

Relative humidity is at all temperatures and pressures defined as the ratio of the water vapour pressure to the saturation water vapour pressure (over water) at the gas temperature:

$$RH = P_w / P_{ws} * 100\% \quad (1)$$

The total pressure does not enter the definition. Above 100°C the same definition is valid. But as the saturation vapour pressure  $P_{ws}$  is greater than 1013 hPa (normal ambient pressure) the RH can't reach 100% in an unpressurised system.

Below 0°C the definition is also valid. Here 100%RH is also impossible because condensation will occur at a lower humidity than 100% (when the vapour is saturated against ice).

## 2. Water vapour saturation pressure

The following formula gives the water vapour saturation pressure to sufficient accuracy between 0°C and 373°C ( W. Wagner and A. Pruß:" The IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use ", Journal of Physical and Chemical Reference Data, June 2002 ,Volume 31, Issue 2, pp. 387535):

$$\vartheta = 1 - \frac{T}{T_c} \quad (2)$$

$$\ln\left(\frac{P_{ws}}{P_c}\right) = \frac{T_c}{T} \left( C_1\vartheta + C_2\vartheta^{1.5} + C_3\vartheta^3 + C_4\vartheta^{3.5} + C_5\vartheta^4 + C_6\vartheta^{7.5} \right) \quad (3)$$

**T** = temperature in K

**P<sub>ws</sub>** = Saturation vapor pressure (hPa)

**T<sub>c</sub>** = Critical temperature, 647.096 K

**P<sub>c</sub>** = Critical pressure 220640 hPa

**C<sub>i</sub>** = coefficients,

**C<sub>1</sub>** = -7.85951783

**C<sub>2</sub>** = 1.84408259

**C<sub>3</sub>** = -11.7866497

**C<sub>4</sub>** = -22.6807411

**C<sub>5</sub>** = -15.9618719

**C<sub>6</sub>** = 1.80122502

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Correspondingly the saturation water vapour pressure over ice at temperatures between -100 and 0.01°C can be calculated using:

$$\theta = \frac{T}{T_n} \quad (4)$$

$$\ln\left(\frac{P_{wi}}{P_n}\right) = a_0(1 - \theta^{-1.5}) + a_1(1 - \theta^{-1.25}) \quad (5)$$

**T** = temperature in K

**P<sub>n</sub>** = Vapor pressure at triple point temperature , 6.11657 hPa

**T<sub>n</sub>** = Triple point temperature 273.16 K

**P<sub>wi</sub>** = Saturation vapor pressure (hPa)

**a<sub>i</sub>** = coefficients,

**a<sub>0</sub>** = -19.928169

**a<sub>1</sub>** = 34.707823

If lower accuracy or a limited temperature range can be tolerated a simpler formula can be used for the water vapour saturation pressure over water (and over ice):

$$P_{ws} = A \cdot 10^{(m \cdot T / (T + T_n))} \quad (\text{hPa}), \text{ where} \quad (6)$$

A, m, T<sub>n</sub> = constants see table 1

**T** = Temperature (°C)

Temperature range(°C)	A	m	Tn	max error
over water:				
-20 ... 50	6.1162	7.5892	240.71	0.09%
50 ... 100	5.9987	7.3313	229.1	0.01%
100 ... 150	5.8493	7.2756	225	0.01%
150 ... 200	6.2301	7.3033	230	0.01%
200 ... 350	10.20614	7.364473	262.76	0.59%
0 ... 200	6.0964	7.33354	230.5	0.38%
Over ice:				
-70 ... 0	6.1134	9.7911	273.47	0.05%

Table 1. Constants for formula 6.

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### 3. Calculation of dewpoint from RH

Calculate  $P_{ws}$  using formula (3) or (6)

Calculate  $P_w = P_{ws} \cdot RH/100$  (in hPa!)

Calculate Dewpoint using formula (7):

$$Td = \left[ \frac{T_n}{\frac{m}{10 \log \left( \frac{P_w}{A} \right)} - 1} \right] \quad (7)$$

The constants in formula (7) are the same as for formula (6).

#### **Example:**

The ambient temperature is 40 °C and the RH is 50%. Calculate  $T_d$ :

$$P_w = P_{ws}(40^\circ\text{C}) \cdot 50/100 = 36.88 \text{ hPa}$$

$$T_d = 237.3 / (7.5 / 10 \log(36.88/6.1078) - 1) = \underline{27.6^\circ\text{C}}$$

### 3.1 Calculation of dewpoint at different pressure

1. Calculate  $P_{ws}$  using formula (3) or (6)

2. Calculate measured vapor pressure (at measurement pressure)

$$P_{w_{meas}} = P_{ws} \cdot \frac{RH}{100} \quad (8)$$

3. Multiply with pressure ratio:

$$P_{w_{pres}} = \frac{P_{pres}}{P_{meas}} \cdot P_{w_{meas}} \quad (9)$$

4. Calculate Dewpoint at process pressure:

$$Td_{pres} = \left[ \frac{T_n}{\frac{m}{10 \log \left( \frac{Pw_{pres}}{A} \right)} - 1} \right] \quad (10)$$

## 3.2 Calculation of RH from dewpoint and ambient temperature

$$RH = 100\% \cdot \frac{Pws(Td)}{Pws(Tambient)} \quad (11)$$

Pws may be calculated using formulas (3) or (6). If the simplified formula (6) is used the expression for RH may be further simplified to:

$$RH = 100\% \cdot 10^{m \left[ \frac{Td}{Td+Tn} - \frac{Tambient}{Tambient+Tn} \right]} \quad (12)$$

Values for the constants M and Tn for the appropriate temperature range can be found in table 1.



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## 4. Using psychrometers

The drybulb-temperature  $T_{\text{dry}}$  and the wetbulb-temperature  $T_{\text{wet}}$  can be converted into  $P_w$  using formula (13):

$$P_w = P_{ws}(T_{\text{wet}}) - P_{\text{tot}} * K * (T_{\text{dry}} - T_{\text{wet}}), \text{ where} \quad (13)$$

$P_{ws}$  = Water vapour saturation pressure from formula (6)

$P_{\text{tot}}$  = Total ambient pressure

$K$  = Psychrometer constant  $0.000662 \text{ } ^\circ\text{C}^{-1}$

When  $P_w$  is known RH can be calculated using (1) or  $T_d$  can be calculated using (7)

### Example:

$T_{\text{wet}}$  is  $38.5^\circ\text{C}$ ,  $T_{\text{dry}}=40.0^\circ\text{C}$  and the ambient pressure is 1013 hPa. Calculate RH and  $T_d$ :

$$P_{ws}(38.5^\circ\text{C})=68.05 \text{ hPa}$$

$$P_{ws}(40.0^\circ\text{C})=73.75 \text{ hPa}$$

$$P_w=68.05 - 1013*0.000662*(40.0-38.5)=67.04 \text{ (hPa)}$$

$$\text{RH}=67.04/73.75*100 =90.9\%$$

$$T_d=237.3/(7.5/^{10}\log(67.04/6.1078))-1=\underline{38.22^\circ\text{C}}$$

## 5. Mixing ratio

The mixing ratio (mass of water vapour/mass of dry gas) is calculated using (14):

$$X = B * P_w / (P_{tot} - P_w) \quad [g/kg] \quad , \text{where} \quad (14)$$

$$B = 621.9907 \text{ g/kg}$$

The value of B depends on the gas. 621.9907 g/kg is valid for air.

In general the constant can be calculated using:

$$B = M(h_2o) / M(gas) * 1000 \quad [g/kg] \quad , \text{where} \quad (15)$$

$M(h_2o)$  = molecular weight of water

$M(gas)$  = molecular weight of gas

For instance for hydrogen we get

$$B = 18.015 / 2.016 * 1000 = 8936 \text{ g/kg}$$

### Example:

The dewpoint  $T_d$  is 40°C and the total ambient pressure  $P_{tot}$  is 998 hPa. Calculate mixing ratio:

$$P_w = P_{ws}(40^\circ C) = 73.75 \text{ hPa}$$

$$X = 621.9907 * 73.75 / (998 - 73.75) = \underline{49.63 \text{ g/kg}}$$

(To obtain the mixing ratio in units of grains/pound use  $B = 4354$  [grains/pound])

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## 6. Enthalpy

Enthalpy can be calculated from mixing ratio using (16):

$$h = T * (1.01 + 0.00189X) + 2.5X \quad (\text{kJ/kg}) \quad , \text{where} \quad (16)$$

T = temperature (°C)

X = mixing ratio (g/kg)

To convert to Btu/lb divide by 2.324

Note that a different base temperature for zero enthalpy, 0°F is often used together with the Btu/lb unit.

### **Example:**

The ambient temperature is 20°C and the relative humidity is 50%. Calculate enthalpy:

$$P_w = P_{ws}(20^\circ\text{C}) * 50/100 = 11.69 \text{ hPa}$$

$$X = 621.9907 * 11.69 / (1013 - 11.69) = 7.26 \text{ g/kg}$$

$$h = 20 * (1.01 + 0.00189 * 7.26) + 2.5 * 7.26 = \underline{38.62 \text{ kJ/kg}}$$

## 7. Absolute humidity

Absolute humidity is defined as the mass of water vapour in a certain volume. If ideal gas behaviour is assumed the absolute humidity can be calculated using (17):

$$A = C \cdot P_w / T \quad (\text{g/m}^3) \quad , \text{where} \quad (17)$$

$C$  = constant 2.16679 gK/J

$P_w$  = vapour pressure in Pa

$T$  = temperature i K

### Example:

The ambient temperature is 20°C and the relative humidity is 80%. Calculate absolute humidity:

$$P_w = P_{ws}(20^\circ\text{C}) \cdot 80/100 = 18.7 \text{ hPa}$$

$$A = 2.16679 \cdot 1870 / (273.16 + 20) = \underline{\underline{13.82 \text{ g/m}^3}}$$

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## 8. Parts per million (ppm)

Parts per million values are usually given vs. the amount of dry air:

I: Volume/volume PPM<sub>v</sub>(dry):

$$PPM_v = \frac{P_w}{P_{tot} - P_w} 10^6 \quad (18)$$

Where

P<sub>w</sub> = water vapour pressure

P<sub>tot</sub> = total pressure

II: Mass/mass PPM<sub>m</sub>(dry)

$$PPM_m = \frac{M_w P_w}{M_d (P_{tot} - P_w)} 10^6$$
$$\frac{M_w}{M_d} = 0.62199 \quad (19)$$

Where

P<sub>w</sub> = Water vapour pressure

P<sub>tot</sub> = Total pressure

M<sub>w</sub> = Molecular mass of water

M<sub>d</sub> = Molecular mass of dry air

From wet air:

III: Volume/volume PPM<sub>v</sub>(wet):

$$PPM_v = \frac{P_w}{P_{tot}} 10^6 \quad (20)$$

#### IV: Mass/mass PPM<sub>m</sub>(wet)

$$PPM_m = \frac{M_w P_w}{M_d P_{tot}} 10^6$$

$$\frac{M_w}{M_d} = 0.62199 \quad (21)$$

#### **Example:**

The dewpoint is 7°C and the total pressure is 998 hPa calculate PPM<sub>v</sub>(dry).

By using formula (6) we get P<sub>ws</sub>(7°C)=10.02 hPa. Now

$$PPM_v = \frac{10.02}{(998 - 10.02)} 10^6 = 10142$$

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## 9. Enhancement factor

The water vapour saturation pressures described in paragraph 2 are exactly valid only in vacuum (water vapour is the only gas present). If other gases are present the real saturation vapour pressure  $P_{ws}$  will increase. For ambient pressure and lower pressures this effect is typically ignored. But at pressures significantly above ambient it has to be taken into account. The enhancement factor  $f$  has been defined as follows:

$$f = \frac{X_w * P}{P_{ws}} \quad , \text{ where} \quad (22)$$

$X_w$  = The mole fraction of water vapour in the gas phase

$P$  = Total pressure

$P_{ws}$  = The saturation water vapour pressure (from paragraph 2)

$x_w * P$  is in effect the water vapour pressure at saturation at pressure  $P$

For CO<sub>2</sub>-free air the following equation for  $f$  has been proposed (Greenspan: J. of Research of the NBS vol 80A, No. 1 p 41-44)

$$f = e^{\left[ \alpha \left( 1 - \frac{P_{ws}}{P} \right) + \beta \left( \frac{P}{P_{ws}} - 1 \right) \right]} \quad (23)$$

The parameters  $\alpha$  and  $\beta$  depend on the temperature as follows:

$$\alpha = \sum_{i=1}^4 A_i t^{(i-1)} \quad (24)$$

$$\beta = e \sum_{i=1}^4 [B_i t^{(i-1)}] \quad (25)$$

$T$  = Temperature (°C)

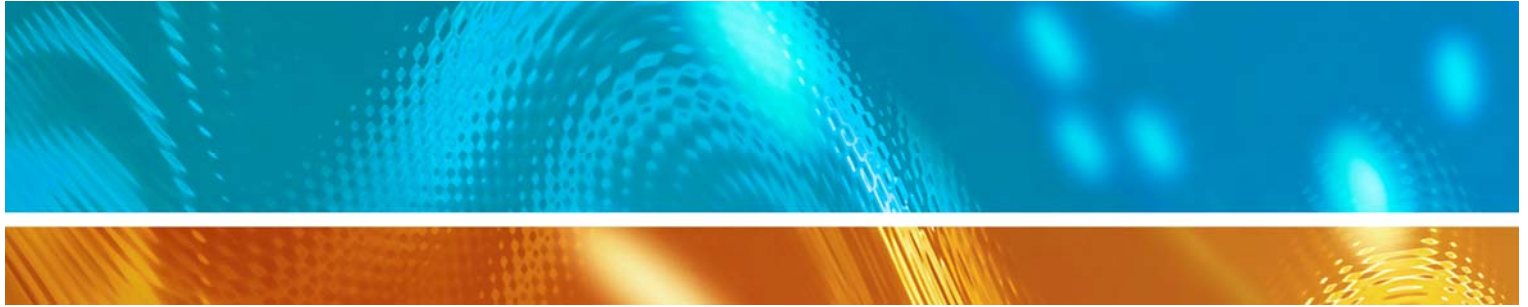
the parameters  $A_i$  and  $B_i$  are listed below:

	water -50 to 0°C	water 0 to 100°C	ice -100 to 0°C
A1	3.62183X10-4	3.53624X10-4	3.64449X10-4
A2	2.60553X10-5	2.93228X10-5	2.93631X10-5
A3	3.86501X10-7	2.61474X10-7	4.88635X10-7
A4	3.82449X10-9	8.57538X10-9	4.36543X10-9
B1	-10.7604	-10.7588	-10.7271
B2	6.39725X10-2	6.32529X10-2	7.61989X10-2
B3	-2.63416X10-4	-2.53591X10-4	-1.74771X10-4
B4	1.67254X10-6	6.33784X10-7	2.46721X10-6

The formulas above are fitted for the pressure range 1 ... 20 atm.

for instance at 20 °C and 10 bars  $f=1.0312$





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